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14. ABSTRACT This report results from a contract tasking Heriot-Watt University as follows: The contractor will 1) determine the conditions for construction of a null sensor using the phase-diversity principle and implemented using a diffractive optical element (DOE) and a single detector plane; 2) investigate and, if possible, quantify necessary conditions for the use of a null sensor based on phase-diversity and the use of DOEs; 3) investigate whether a null sensor based on phase diversity and DOEs can be used to separate isoplanatic and anisoplanatic wavefront distortions.					
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Compact adaptive optics systems II (CAOS-II)

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Final Report covering 22 Sept 02 to 21 Sept 03

Abstract

This study has successfully confirmed and generalised sufficient conditions for construction of a null wavefront sensor based on the principles of a generalisation of phase-diversity wavefront sensing. This confirmation validates analytical results from a previously-funded study. In the present study we have also established the necessary conditions for such a sensor. Validation of this theoretical work has been conducted using computer simulation and evidence that the generalised wavefront sensor can be optimised to exploit *a priori* information in order to maximise sensitivity has been demonstrated.

1. Background

This programme commenced on 22 September 2002 and was proposed as a continuation of F61775-01-W-E063, the final report on which was submitted in August 2002. Progress reports covering the first, second and third quarters have previously been submitted. This final report provides a summary of progress during the previous quarters and details the progress during the final quarter from 22nd July to 21st September 21 2003.

This programme of work aimed to confirm and extend the theoretical work from the previous study¹.

The objectives stated in the proposal for this second phase of study were stated as²:

- To confirm and generalise *sufficient* conditions for construction of a null sensor using the phase-diversity principle and implemented using a diffractive optical element (DOE) and a single detector plane
- To investigate and, if possible, quantify *necessary* conditions for the use of a null sensor based on phase-diversity and the use of DOEs
- To investigate whether a null sensor based on phase diversity and DOEs can be used to separate isoplanatic and anisoplanatic wavefront distortions.

2. General Scheme

The CAOS concept is a compact adaptive optical system based on transparent wavefront modulators (e.g. liquid crystal SLMs) and phase-diversity wavefront sensing.

The rationale for CAOS is that a compact Adaptive Optical (AO) system, based on the use of DOEs, phase-diverse wavefront sensing and transparent wavefront modulators, offers the potential for construction of AO systems with benefits such as:

1. Minimisation of non common-path errors by combining the wavefront sensor data and the corrected image in a single focal plane with essentially no separation of the 'science' and wavefront sensing optical trains
2. The ability of place wavefront modulators conjugate to multiple planes in the object space whilst preserving a compact and robust optical train that would be particularly well-suited to multi-conjugate AO (MCAO)
3. Avoidance of the requirement for conventional optics, such as beam splitters and conventional bulk optical components for re-imaging conjugate planes, that increase the size and weight of the equipment.

¹ Final Report on Agreement Number F61775-01-W-E063, 4 August 2001.

² Proposal *Compact adaptive optics systems II (CAOS-II)*, submitted 26 August 2002.

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A schematic representation of a compact adaptive optical system according to one realisation of the CAOS concept is shown in figure 1.

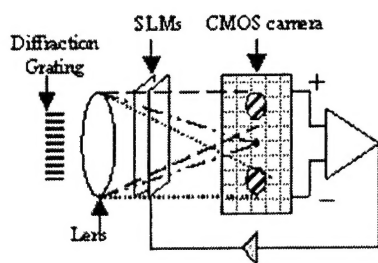


Figure 1: A schematic of a suggested compact AO system.

Figure 1 Compact AO System (CAOS) concept. A DOE is used (with the appropriate filter function) to produce the phase diverse data and the image of the source onto a CMOS camera. We suggest the use of SLM's (such as liquid crystals) for easily controlled modulation of the wavefront. In this design we have, as far as is possible, made the propagation common path as we believe that this is the easiest way to build a compact system.

Figure 2, below, illustrates the CAOS wavefront sensing concept. An imaging system is constructed with a diffractive optical element (DoE) positioned in the plan of the imaging lens. The DoE generates various diffraction orders $0, \pm 1, \pm 2, \dots$. By adjusting the phase amplitude, the DoE can be engineered to ensure that most of the radiation is concentrated into the orders 0 and ± 1 , thus higher diffraction orders will be ignored in the following discussion. Under some circumstances the target field may represent an image plane, in which case the zero diffraction order contains the AO-corrected image of the scene. In other cases the target field may represent to input pupil to an optical system and the zero order may then represent the input to an imaging system, hyperspectral imager, spectrograph, etc.

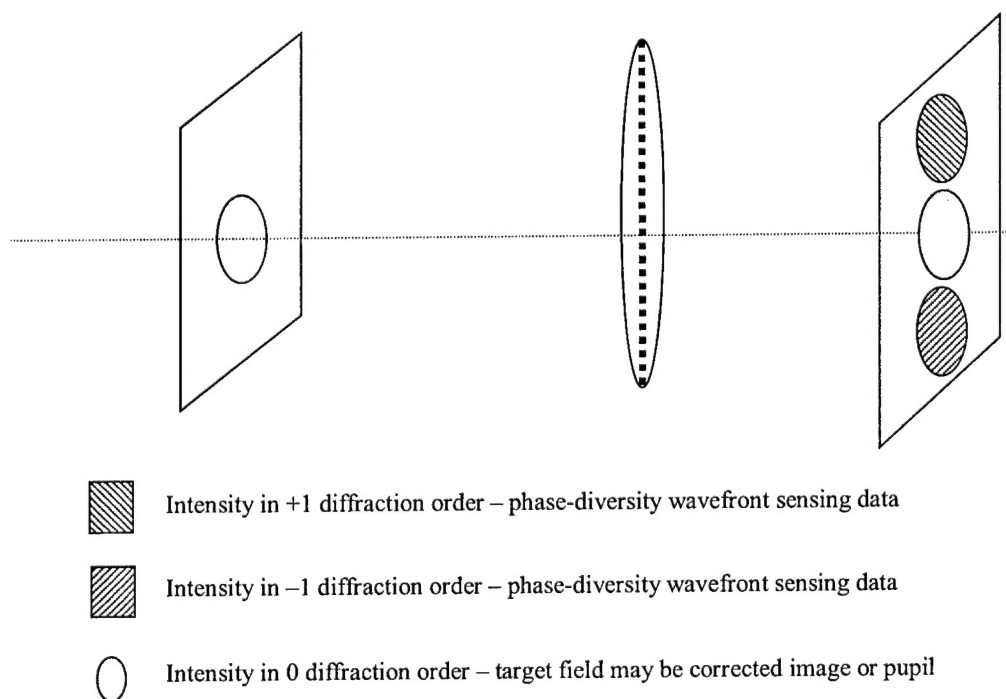


Figure 2: Schematic CAOS wavefront sensing concept.

Previous wavefront sensing work with such systems has employed DoEs designed using curved gratings³. Such curved gratings provide a defocus change whose arithmetic sign changes with the sign of the

³ Blanchard P M, Fisher D J, Woods S C and Greenaway A H, *Phase-diversity wavefront sensing using a distorted diffraction grating*, *Applied Opt* 39(2000)6649-6655

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diffraction order and have been shown to provide simultaneous multi-plane images^{3,4} suitable as input data for conventional phase-diversity wavefront reconstruction.

The conventional approach to phase-diverse wavefront sensing relies on the solution to the differential Intensity Transport Equation (ITE) and this formulation imposes several constraints on the nature of the wavefront to be reconstructed⁵. From the point of view of the current work, the important restrictions imposed are that:

- The wavefront phase reconstructed must be everywhere continuous within the pupil
- The derivative of the wavefront phase must be everywhere continuous within the pupil
- The wavefront reconstruction requires computing effort and occasions delay, whereas it may be possible to derive a signal that can drive the wavefront modulator directly from the wavefront sensor data.

These restrictions are important, in particular the first two potentially exclude the use of the pixellated liquid crystal wavefront modulators that offer one of the most attractive options for cheap, low-mass, low-volume and low-voltage wavefront modulators. The third restriction reduces the frequency with which the AO control loop can be closed compared to use of a null sensor. Further the presence of zero intensity points within the wavefront leads to logarithmic singularities (branch points) which imply discontinuities in the phase function. This implies that the wavefront reconstruction will be poor when the input represents scintillated wavefronts.

The CAOS programme was intended to investigate alternative formulations of the phase-diversity wavefront sensing principle in anticipation that solutions may be found that are more compatible with operation as a null wavefront sensor suitable for use with wavefronts whose phase may be discontinuous or whose derivative may be discontinuous.

In the previous realisation of Phase-diverse wavefront sensing (PDWFS) the reconstructed wavefront phase is flat iff (if and only if) the difference between the images in the +1 and -1 diffraction orders (see figure 2) is zero. Essentially, the two images represent forward and backward propagation of the wavefront from the plane in which the wavefront phase is to be characterised (the reference plane). The forward and backward propagation from this plane can yield identical results (i.e. equal image intensities) iff the wavefront in the reference plane can be represented by a real function. For our purposes here* a real wavefront may be characterised by any wavefront whose phase is uniform valued. If the wavefront is curved, the propagation of a concave (convex) wavefront leads to a converging (diverging) beam with a consequent increase (decrease) of the axial intensity. Thus the intensity on the two recording planes can be equal iff the input wavefront is not curved and, as we shall see, is not otherwise distorted.

For measurements designed to measure wavefront phase in the input pupil of an imaging system these concepts can be represented schematically by figure 3. In this figure the wavefront on the central plane in the cylinder is to be reconstructed from intensity measurements made on discs representing the two ends of the cylinder.

If we discard the physical picture provided by figure 3, we can regard the images formed in the ± 1 diffraction orders (figure 2) as representative of the wavefront present in the centre (reference) plane convolved with an aberration function that has equal but opposite aberration in the \pm diffraction orders. In the case where the aberration is pure defocus the data is amenable to a simple physical interpretation involving propagation through a cylinder whose ends are the planes on which the two intensities are measured. In the case where the aberration is not defocus this simple physical picture is no longer appropriate.

Several obvious questions follow from this re-posing of the problem:

- What, if anything, is unique about the use of defocus as the aberration function?
- What generic properties should an aberration function suitable for use in wavefront sensing possess?

⁴ Blanchard P M and Greenaway A H, Simultaneous multi-plane imaging with a distorted diffraction grating, App. Opt. **38**(1999)6692-6699.

⁵ Woods S C and Greenaway A H, Wavefront sensing using a Green's function solution to the intensity transport equation, J Opt Soc Am A **20**(2003)508-512.

* The phase of a real wavefront can be zero or π , the latter corresponding to a negative real value. This is unimportant here but potentially important when using the wavefront sensor in metrology applications.

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- Can the aberration function used be optimised to maximise sensitivity through the use of *a priori* information about the wavefront to be measured?

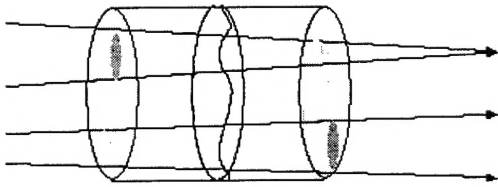
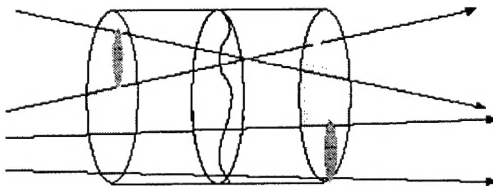


Figure 3:

3a) A concave wavefront propagating from left to right leads to a local increase of intensity. A convex wavefront leads to a decrease in the local intensity.

In each case here the intensities recorded in the ± 1 diffraction orders represent the intensities on the ends of the cylinder.



3b) Ambiguity can arise if the curvature is so severe that a focus point occurs within the volume sampled by the measurements. The upper part of each figure leads to the same intensity increase

In the following sections we will address the first two of these questions.

3. Description of PDWFS data

The preliminary study¹ concluded that '... defocus aberration is only one of a set of aberration functions that can be used to give phase-diverse wavefront sensors that can be operated as a null sensor'. Given that other aberrations can be employed an obvious next step, essential if any optimisation is to be attempted, is to quantify necessary and sufficient conditions for a particular aberration function to be useful for wavefront sensing. Because CAOS is intended to investigate possible schemes leading to a compact adaptive optics systems, attention has been restricted to the situation where equal and opposite sign aberrations have been employed – this is compatible with the use of simple DoE elements. The use of asymmetric aberrations is neither excluded nor covered by the following analysis.

The following analysis formed the basis for a paper presented⁶ at CLEO 2003 in Munich on 24 June 2003 and represents extracts from a draft paper in preparation for submission to a peer-reviewed journal. As advised in the progress report for the third quarter and after discussion with EOARD, the intellectual property represented by this analysis was the subject of a UK patent application⁷.

Let $\Psi(r) = |\Psi(r)|e^{i\varphi(r)}$ represent the complex amplitude distribution in the entrance pupil of an optical system, r being the co-ordinate in the pupil plane. If $\Psi(r)$ represents a plane wavefront, the phase satisfies $\varphi(r) = \text{constant}$. Without loss of generality this constant may be taken to be zero. Any spatial variation in the wavefront phase represents a distortion, or aberration, that requires correction in an AO system.

To operate successfully as a null wavefront sensor in an AO system we require a device that produces an error signal if, and only if, $\varphi(r)$ is not constant*. It is desirable that the error signal provides

⁶ Campbell H I, Greenaway A H and Restaino S R, *Compact Adaptive Optics*, CLEO2003.

⁷ *Novel Wavefront Sensor*, reference GB0314444.1, priority date 20 June 2003

* We will consider the optical output from monochromatic systems here. Thus a discontinuity of exactly an integral number of wavelengths in size does not affect the optical output, e.g. image intensity profile, and may be regarded as an undistorted wavefront. Such a discontinuity becomes a potentially-important wavefront feature in metrology applications.

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information that localises the wavefront error in r -space and indicates the sense in which correction should be effected (in the absence of such indication a multi-dither technique is required to effect correction).

Let $\psi(\xi) = H(\xi) + A(\xi)$ be a filter function whose Fourier transform, $\Psi(r)$, is the aberration function with which the input wavefront is convolved. $H(\xi)$ and $A(\xi)$ represent respectively the Fourier transforms of the real and imaginary parts of $\Psi(r)$. Clearly, $H(\xi)$ is Hermitian and $A(\xi)$ is anti-Hermitian. These symmetry properties will be required later. Thus,

$$\begin{aligned} H(\xi) &= H^*(-\xi) \\ A(\xi) &= -A^*(-\xi) \end{aligned} \quad (1)$$

Let $F_{\pm}(\xi) = R(\xi) \pm iI(\xi)$ be filter functions whose Fourier transforms, $f_{\pm}(r)$, which represents aberration functions with which $\Psi(r)$ is convolved when recording phase-diverse data. The functions $R(\xi)$ and $I(\xi)$ are real-valued functions and the \pm indicates the use of two filter functions, in which the phase of the filter function is reversed. In this work we are particularly interested in the necessary and sufficient conditions that constrain $R(\xi)$ and $I(\xi)$ in such a way that F_{\pm} are suitable filter functions to provide a null wavefront sensor for use in adaptive optics.

The detected phase-diversity intensity functions may thus be written

$$j_{\pm}(r) = \left| \int d\xi \psi(\xi) F_{\pm}(\xi) e^{-i\xi \cdot r} \right|^2. \quad (2)$$

Substituting for F_{\pm} , expanding and simplifying, the difference between the images formed using the two filter functions may be expressed

$$\begin{aligned} d(r) &= j_+(r) - j_-(r) \\ &= 2i \left[\int d\xi \psi(\xi) I(\xi) e^{-i\xi \cdot r} \int d\xi' \psi^*(\xi') R(\xi') e^{i\xi' \cdot r} \right. \\ &\quad \left. - \int d\xi \psi(\xi) R(\xi) e^{-i\xi \cdot r} \int d\xi' \psi^*(\xi') I(\xi') e^{i\xi' \cdot r} \right] \end{aligned} \quad (3)$$

This is a real-valued function, since the quantity in [] is a difference of two complex conjugates and is thus imaginary-valued, so

$$\begin{aligned} \frac{d(r)}{2i} &= \int d\xi [H(\xi) + A(\xi)] I(\xi) e^{-i\xi \cdot r} \int d\xi' [H^*(\xi') + A^*(\xi')] R(\xi') e^{i\xi' \cdot r} \\ &\quad - \int d\xi [H(\xi) + A(\xi)] R(\xi) e^{-i\xi \cdot r} \int d\xi' [H^*(\xi') + A^*(\xi')] I(\xi') e^{i\xi' \cdot r}. \end{aligned} \quad (4)$$

The rhs of (4) can then be expanded and the terms grouped into 4 separate expressions, which are equal to the rhs of (4) when summed:

$$\int d\xi H(\xi) I(\xi) e^{-i\xi \cdot r} \int d\xi' H^*(\xi') R(\xi') e^{i\xi' \cdot r} - \int d\xi H(\xi) R(\xi) e^{-i\xi \cdot r} \int d\xi' H^*(\xi') I(\xi') e^{i\xi' \cdot r} \quad (5.1)$$

$$\int d\xi H(\xi) I(\xi) e^{-i\xi \cdot r} \int d\xi' A^*(\xi') R(\xi') e^{i\xi' \cdot r} - \int d\xi A(\xi) R(\xi) e^{-i\xi \cdot r} \int d\xi' H^*(\xi') I(\xi') e^{i\xi' \cdot r} \quad (5.2)$$

$$\int d\xi A(\xi) I(\xi) e^{-i\xi \cdot r} \int d\xi' H^*(\xi') R(\xi') e^{i\xi' \cdot r} - \int d\xi H(\xi) R(\xi) e^{-i\xi \cdot r} \int d\xi' A^*(\xi') I(\xi') e^{i\xi' \cdot r} \quad (5.3)$$

$$\int d\xi A(\xi) I(\xi) e^{-i\xi \cdot r} \int d\xi' A^*(\xi') R(\xi') e^{i\xi' \cdot r} - \int d\xi A(\xi) R(\xi) e^{-i\xi \cdot r} \int d\xi' A^*(\xi') I(\xi') e^{i\xi' \cdot r} \quad (5.4)$$

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This expression for the difference between the two detected intensity functions is generally valid – no restricting assumptions have so far been made. We may now investigate under what conditions of symmetry these expressions individually, or summed, are identically zero, i.e. the conditions under which the difference between the two images might be useful as a null wavefront sensor.

4. Necessary and Sufficient conditions for null sensing of the wavefront

a. Filter function must be complex-valued

If either R or I are non-zero $\forall \xi$, all of the terms in equations (5) are identically zero $\forall \psi$. In this case $d(r)$ is identically zero for all input wavefronts and no error signal is generated from a non-flat wavefront. Thus filter functions that are purely real or purely imaginary are not acceptable filters for use in producing a null wavefront sensor.

b. Filter functions with complex-values

For filter functions with complex values we need to consider the odd or even symmetry of both the real and imaginary part of the filter function.

b1. Even symmetry

Suppose that both $I(\xi)$ and $R(\xi)$ are even functions of ξ .

Consider the first expression (5.1). Since $H(\xi)$ is Hermitian and $I(\xi)$ is symmetric and real-valued the product $H(\xi)I(\xi)$ is Hermitian. Thus the Fourier integral $\int d\xi H(\xi)I(\xi)e^{-ir\xi}$ is real-valued.

The same is true of $\int d\xi' H^*(\xi')R(\xi')e^{ir\xi'}$. The second product of two integrals is term by term the complex conjugate of the first product. Thus expression (5.1), the difference between two complex conjugates, is always zero when both $I(\xi)$ and $R(\xi)$ are symmetric.

Similarly (5.4) is always zero because each of the integrals reduces to a purely imaginary function. The product of these imaginary functions is real and the difference between the two complex conjugate terms is again always zero.

Thus the difference between the two images is the sum of (5.2) and (5.3) and can be written

$$\begin{aligned} \frac{d(r)}{2i} = & \int d\xi H(\xi)I(\xi)e^{-ir\xi} \int d\xi' A^*(\xi')R(\xi')e^{ir\xi'} - \int d\xi A(\xi)R(\xi)e^{-ir\xi} \int d\xi' H^*(\xi')I(\xi')e^{ir\xi'} \\ & + \int d\xi A(\xi)I(\xi)e^{-ir\xi} \int d\xi' H^*(\xi')R(\xi')e^{ir\xi'} - \int d\xi H(\xi)R(\xi)e^{-ir\xi} \int d\xi' A^*(\xi')I(\xi')e^{ir\xi'} \end{aligned} \quad (6)$$

In equation (6), for each pair of integrals one integral reduces to a real-valued function and the other integral to an imaginary-valued function. The first and second terms, also the third and fourth terms, are complex conjugate pairs and thus the rhs of (6) is imaginary valued or zero.

If either H or A is zero or if $A = He^{i\varphi}$ with φ constant, equation (6) and thus $d(r)$, is zero.

However, these conditions are exactly those under which the input wavefront is flat and a null sensor is required to produce a null output.

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For equation (6) to give a zero result for other input wavefronts, $\Psi(r)$, it is necessary that equations (5.2) and (5.3), both of which are imaginary valued, should have imaginary parts that are equal and opposite in sign. For non-trivial inputs this is true iff $I(\xi)$ and $R(\xi)$ are scaled versions of each other, in which case the filter function can be represented as purely real or purely imaginary and the considerations of section a. apply.

Thus, if the filter function is complex with even symmetry the difference between the two images formed using these filters forms a potentially-useful null wavefront sensor. Note that the defocus filter can be expressed $F_{\pm}(\xi) = \exp(\pm i\Delta\xi^2)$, a function with symmetric but non-scaled real and imaginary parts and one that thus fits the description given in this section.

b2. Odd symmetry

Suppose that both $I(\xi)$ and $R(\xi)$ are odd functions of ξ .

In (5.1) and in (5.4) the odd symmetry of the real functions R and I means that the products within the integrals in (5.1) have anti-Hermitian symmetry and those within (5.4) have Hermitian symmetry. Thus, although the arguments given are reversed for each term from those given in section b1, both (5.1) and (5.4) are identically zero $\forall \psi$. Thus $d(r)$ again reduces to equation (6). The arguments from section b1 again hold, although the role of the terms is reversed, one term in each integral product is purely real and the other term purely imaginary.

Thus a filter function with complex odd symmetry is potentially suitable for use as a filter function for wavefront sensing.

b3. Mixed symmetry

Suppose that one of $I(\xi)$ and $R(\xi)$ is an even function of ξ and the other is an odd functions of ξ .

In (5.1) the mixed symmetry will result in one of the integrals in each product being purely imaginary and the other integral purely real (dependent on whether I or R is odd). In either case the product of the integrals is purely imaginary and thus (5.1) is purely imaginary or zero. An equivalent argument shows that (5.4) is purely imaginary or zero.

In (5.2) and (5.3) the mixed symmetry results in both integrals in each product being either purely real-valued or purely imaginary-valued. In either case each product reduces to a real-valued function and since the two integral products in each equation are complex conjugate pairs the equations (5.2) and (5.3) are zero $\forall \psi$.

Thus the difference between the two images can be expressed

$$\begin{aligned} \frac{d(r)}{2i} = & \int d\xi H(\xi) I(\xi) e^{-ir\xi} \int d\xi' H^*(\xi') R(\xi') e^{ir\xi'} - \int d\xi H(\xi) R(\xi) e^{-ir\xi} \int d\xi' H^*(\xi') I(\xi') e^{ir\xi'} \\ & + \int d\xi A(\xi) I(\xi) e^{-ir\xi} \int d\xi' A^*(\xi') R(\xi') e^{ir\xi'} - \int d\xi A(\xi) R(\xi) e^{-ir\xi} \int d\xi' A^*(\xi') I(\xi') e^{ir\xi'} \end{aligned} \quad (7)$$

Note, however, that the expressions (5.1) and (5.4) are reliant on the interactions between H and the filter function, or on A and the filter function, and do not involve cross terms between H and A .

In equation (7) the mixture of odd and even symmetry will mean that in each integral product one integral will reduce to a purely real-valued function and the other integral to a purely imaginary-valued function. Thus each integral product will be purely imaginary valued. Since equations (5.1) and (5.2) are the difference of complex conjugates, equation (7) is imaginary valued or zero. For the result to be zero $\forall r$

we note that $\forall r$ at least one integral in each product must be zero. This is not possible with non-trivial inputs, since all terms are entire functions of exponential type. Thus such filters are liable to produce an error signal for any non-trivial input wavefront.

Thus, filters with mixed symmetry produce are unsuitable for use as a wavefront sensor.

5. Sensing the error direction

We have established that the crucial term that encodes information about the wavefront aberrations is the sum of the cross terms in equation (6).

If the sense of the wavefront error is reversed the arithmetic sign of $A(\xi)$ reverses because the phase of the wavefront will change arithmetic sign. This means that, for a wavefront error of a given amplitude, the error signal (equation (6) for filter functions with complex even or complex odd symmetry) changes sign if the sense of the error is reversed. The wavefront sensor thus delivers an error signal that preserves information about the sense of the wavefront error.

Note that, since the relationship between the error signal and the wavefront error is non-linear and periodic with wavelength (involving the balance between H and A in the description of the wavefront), this does not guarantee that the error signal can be inverted to find the wavefront shape. The analysis given does not guarantee that the arithmetic sign (nor indeed the magnitude) of the error can be deduced from the sign (or magnitude) of the error signal.

6. Localising the wavefront error

Returning to equation (6) we note that each of the integrals, when expressed as a function of r , appears in the form of a convolution of the Fourier transform of H or A with a Hermitian function related to the filter function.

Without loss of generality, the location of the wavefront error can be identified with the position at which $a(r)$, the Fourier transform of $A(\xi)$, is non-zero ($a(r)$ is directly related to the phase distortion). The function with which the convolution takes place may or may not have a maximum at the origin, but even when it does not the location of the non-zero component is, in some sense, localised at the point where $a(r)$ is non-zero in the sense that it may be distributed around the position at which $a(r)$ is non-zero.

7. Conditions for null wavefront sensing - summary

Necessary and sufficient conditions for aberration functions suitable for use in a generalised phase-diversity null wavefront sensor have been developed.

These show that a pair of aberration functions in which the aberrations are applied as a complex conjugate pair will satisfy the requirements for null wavefront sensing provided that:

1. The aberration functions have complex Fourier transforms (both real and imaginary parts of the Fourier transform of the aberration point spread must be complex)
2. The real and imaginary parts of the Fourier transform of the aberration function must both have even symmetry or both have odd symmetry (mixed symmetries are not allowed).

For filter functions constructed as described above, the deviation of an input wavefront from a plane wave will produce an error signal that is non-zero for non-flat wavefronts and zero for flat input wavefronts. The analysis also demonstrates that a change in the direction of deviation from a plane wave on the input wavefront will change the arithmetic sign of the error signal (showing that the sensor indicates the sense of the wavefront error). Further, the error signal is, in a mathematical sense, localised to the position of the error on the wavefront.

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We have not demonstrated here that the sensing of the error direction nor the location of the error is always useful for null sensing. However, heuristic experience with the defocus-based phase diversity suggests that both the sense of direction and the location of the error are preserved with sufficient fidelity for the system to be useful as a null sensor.

8. Computer simulations

The computer simulation of the analysis presented here has consisted of two phases. Firstly we have used Mathematica to verify the form of the equations presented above, giving confidence in these results.

In the third progress report we suggested that '...a demonstration of the new sensor, even if only achieved at the simulation level, is the priority for the final quarter of this programme' and that '...little progress will be made in examination of the properties of the wavefront sensor applied under anisoplanatic conditions.' EOARD were invited to comment on this priority and the absence of comment has been taken as indicative of agreement that they were in agreement with this selection.

Computer simulations have been undertaken with a range of input wavefronts and phase-diversity filter functions having the properties identified above. All input wavefronts used were smooth functions. In order to assess the utility of the wavefront sensor an iterative algorithm belonging to the general class of 'error reduction algorithms' has been used. These simulations are not claimed to be to be anything other than illustrative, through the rate of convergence, of the efficacy of various phase-diversity filters for wavefront sensing.

Figure 4 below shows a computer simulation in which a test wavefront is reconstructed using an error-reduction algorithm, comparative results are presented for the same signal to noise and phase diversity functions equal to defocus ($Z_{2,0}$) and equal to $Z_{10,0}$. In each case the upper two plots show false colour representation for the test wavefront and the reconstruction achieved, the central plots show the difference between the test function and the reconstruction achieved (centre, left) and the log error metric against iteration number (centre right). In the lower two plots the input intensity functions are shown. For these two phase diversity functions there is little difference in the rate of convergence or the quality of the solution. The convergence is slightly faster using $Z_{10,0}$ in this example, but that advantage is not always preserved in different examples. The error metric used is

$$Error = \sum_{j=1}^2 \frac{\int dr (I_j(r) - \tilde{I}_j(r))^2}{\int dr I_j^2(r)}$$

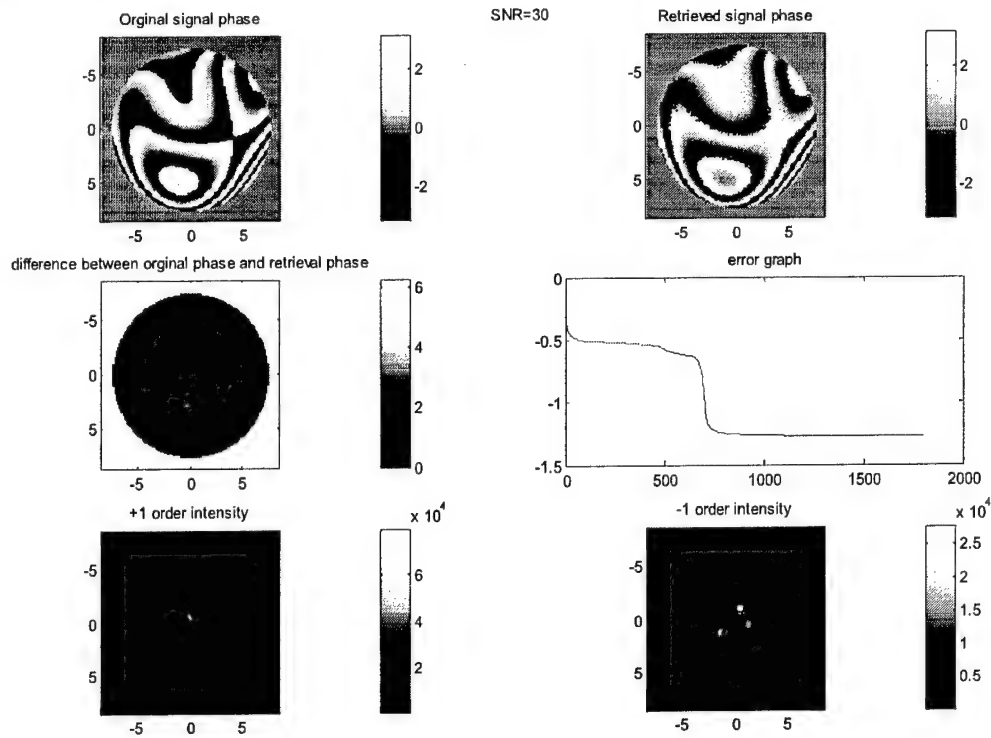
where I_j represents measured intensity data in the ± 1 diffraction orders and \tilde{I}_j represents an estimate of the phase diversity data intensity from the reconstructed wavefront.

In general it has been noted that the use of a phase diversity function other than defocus can be used to optimise the wavefront reconstruction where suitable *a priori* information is available. However, it is also noted that a poor choice of diversity function can lead to poor algorithm performance. Thus the choice of a symmetric phase diversity function appears generally beneficial if there is no *a priori* indication that the input wavefront is likely to be asymmetric. However, the use of astigmatic or other phase diversity functions showing non-symmetric behaviour has been found to offer benefit when reconstructing wavefields that have definite directional asymmetries.

Thus, the use of symmetric phase diversity functions, such as defocus ($Z_{2,0}$), spherical aberration ($Z_{4,0}$), etc appear to offer the best choice when designing a wavefront sensor for application to wavefront sensing problems associated with atmospheric transmission. To our surprise the optimal diversity functions appear to be defocus ($Z_{2,0}$) and $Z_{10,0}$. We can, at present, offer no logical explanation for the apparent superiority of these defocus functions over functions such as spherical aberration. Although the simulation results presented here use phase diversity functions consisting of pure Zernike terms, it should be noted that the analysis presented demonstrates that combinations of terms having suitable symmetry properties are permissible providing that the appropriate symmetries are preserved.

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Diversity filter $2\pi Z_{2,0}$ (defocus)



Diversity filter $2\pi Z_{10,0}$

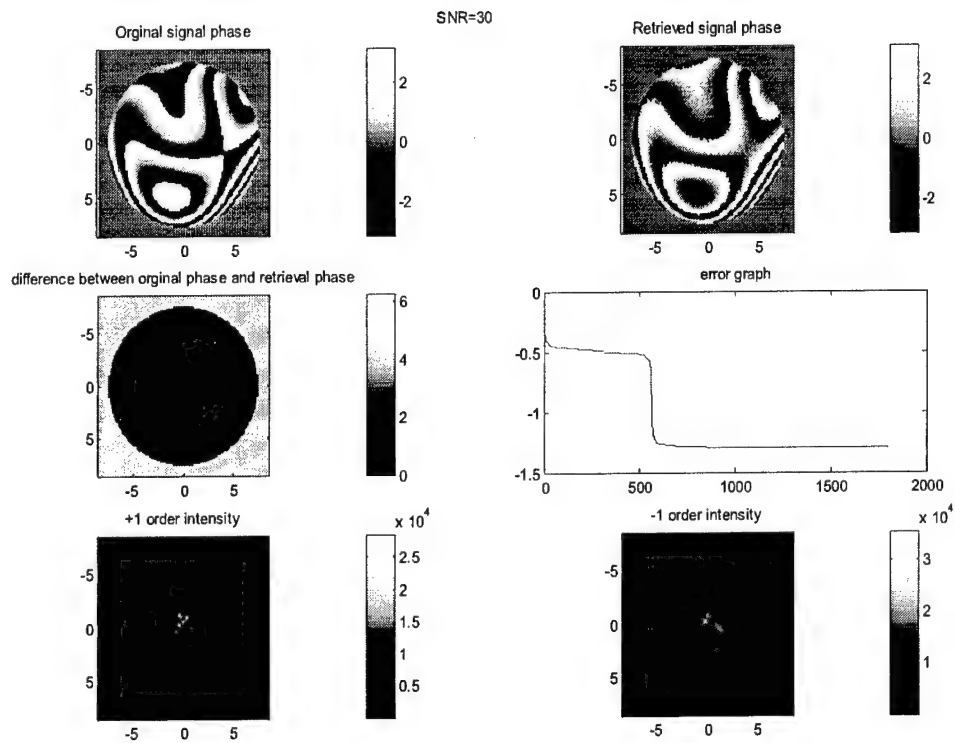


Figure 4 – comparison of iterative solutions using different diversity functions

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The following figures (figures 5, following pages) illustrate the use of diversity filters that are not axially symmetric.

These examples use the same test wavefront as the examples given in figure 4. Simulations have been conducted at various levels of SNR, from noise-free data through to a signal to noise level of 10.

In the examples given here it should be noted that the wavefront reconstructions achieved using astigmatic diversity filters are superior (faster convergence) to those obtained using a defocus-only filter if the axis of the filter is correctly selected and inferior if the axis selected is at 45° to the preferred direction. This superiority is preserved also with noise-free data and results for SNR 30 are shown as well as those for the noise-free case.

This demonstrates a result that might well have been expected, that use of correct *a priori* prejudice can be used advantageously to improve algorithm performance, but that if that prejudice is wrong the algorithm performance is damaged.

In metrology applications of wavefront sensing one might reasonably well expect to have substantial and accurate *a priori* information about the wavefront shape to be determined, based on knowledge of the shape that is desired and the angle and structure of the illuminating source.

However, for applications to sensing of turbulence-induced aberrations the most *a priori* information available is in the form of the statistical description expected from atmospherically-distorted wavefronts. The expected power represented by various Zernike polynomials in turbulence-degraded wavefronts has been evaluated for various turbulence models. It seems probable that *a priori* knowledge of the contribution that various Zernike polynomials make to turbulence-degraded wavefronts can be exploited to design a phase diversity filter in which several Zernike polynomials are combined with weights designed either to reduce sensitivity to high-order terms or preferentially to increase the sensitivity to high order terms.

This latter strategy may offer benefit in terms of high dynamic range imaging through turbulence (e.g. reduction of laser dazzle effects). If one can partition measurement error more evenly between high and low order modes in the sensing of turbulence-induced aberrations it should be possible to make more accurate measurements of the high-order modes. The limit at which these give rise to aberrations at a specific level of impact is known from the statistical models and it is these higher order modes (and not the larger amplitude low order modes) that generally dictate the angular spread of the atmospheric 'seeing' disc.

The determination of the optimum combination of Zernike functions for sensing of wavefront with particular statistics is a significant undertaking and beyond the cope of this study (see Future Work).

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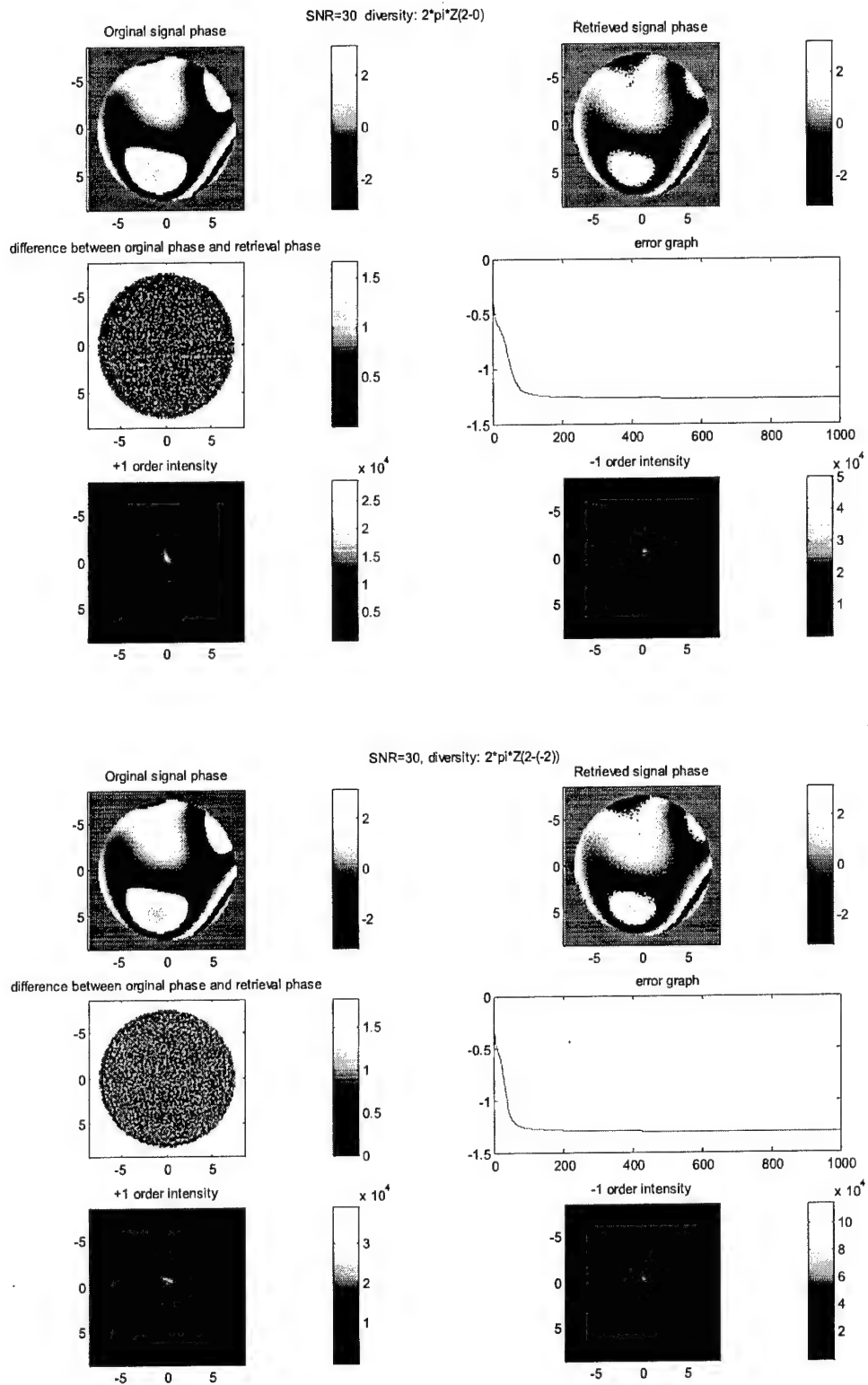


Figure 5: Reconstruction using defocus-only kernel (5a-upper) and using astigmatism (5b-lower). Noisy simulated data with SNR ~30. Continued next

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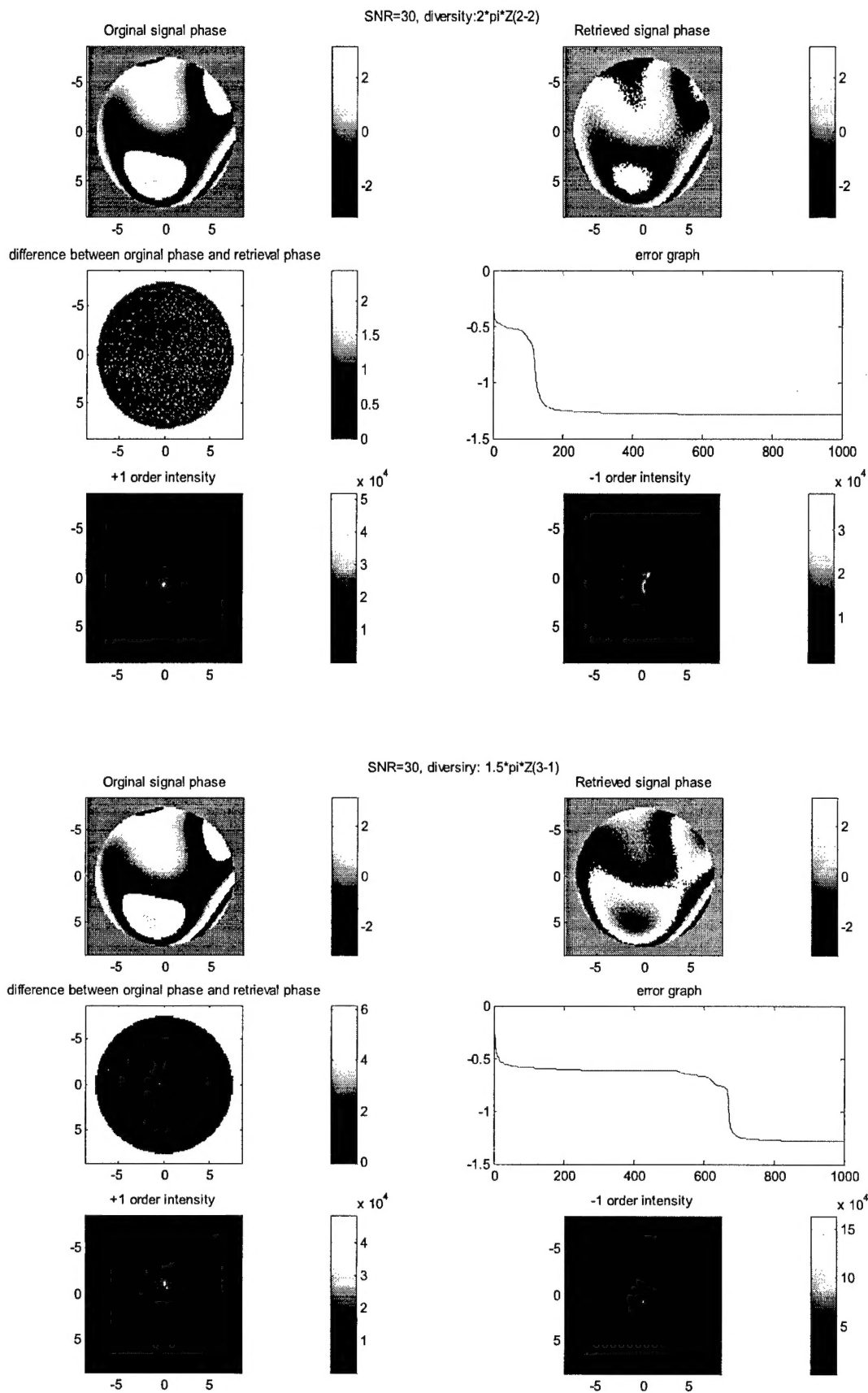


Figure 5: 5c (upper) Reconstruction using kernel with astigmatism rotated by 45° compared to 5b and a kernel with 3rd order coma (5d-lower). SNR~30.

9. Summary and conclusions

Within this programme we have successfully achieved the first set of stated objectives:

- '...to confirm and to generalise *sufficient* conditions for phase diversity filters that can be used for wavefront sensing'.

The sufficient conditions for use of a filter function as a wavefront sensor are that:

- the filter function cannot be represented as a real function by multiplying by any constant phasor
- the complex filter function has similar symmetries in both its real and imaginary parts (i.e. both having even symmetry of both having odd symmetry).

Filters with the symmetry properties described above are manufactured relative easily and at modest cost using diffractive optical elements or could be realised in other ways, e.g. using bulk optical components. Such filters can be used in the manner indicated in figure 1 and are thus compatible with the CAOS concept of compact AO systems. The original plan to make experimental measurements with such filters proved over ambitious when combined with the need to establish IPR protection for the work.

It should be noted, however, that if operation of the AO system is required using a DOE in a configuration where the zero diffraction order contains a corrected image of the scene and the wavefront correction to be achieved using a null sensor, the filter functions used must contain a defocus term (in addition to other diversity filter components or in isolation). The use of a defocus term is necessary to provide a one-to-one mapping between the wavefront sensor data and the mirror actuators.

It should be further noted that for monochromatic wavefronts the dislocation of the wavefront by an integral number of π radians does not produce an error signal.

Within this programme we have, within constraints, successfully achieved the second objective:

- '...to investigate and, if possible, quantify *necessary* conditions for the use of a null sensor based on phase-diversity and the use of DOEs'

We have demonstrated the symmetry conditions described above are *necessary* conditions by showing that filter function possessing other symmetries will produce a null output for all non-trivial input wavefronts.

Note that the treatment given here applied only to pairs of filter functions in which the sign of the filter phase is reversed in the two filter functions used to produce the phase diversity data. This is fully consistent with the use of DOEs to produce the requisite data on a single focal plane. However, the treatment given here does not apply to pairs of filter functions in which another basis is chosen to produce two different phase diversity filters. The necessary conditions therefore only apply strictly within the restriction that the phase of the filter function is reversed between the measurement of the two sets of phase diversity measurements. This imposition of phase reversal is the constraint on the results.

For various reasons, but principally due to diversion of effort in order to file for patent protection on the new filter designs arising from the analysis given here, we undertook no investigation of the third stated objective:

- To investigate whether a null sensor based on phase diversity and DOEs can be used to separate isoplanatic and anisoplanatic wavefront distortions.

In order to demonstrate wavefront sensing using the new filter functions we have undertaken some illustrative computer simulations showing that wavefront reconstruction can be achieved using iterative algorithms based on 'error reduction' type algorithms. A substantial number of computer simulations have been undertaken. These have demonstrated

1. The implied ambiguity in the phase diversity wavefront sensing data when the wavefront is real but has both positive and negative regions.

It was noted in a footnote (page 3) that the analysis given here strictly demonstrated that real-valued functions with both positive- and negative-valued regions satisfy the symmetry constraints exploited in the analysis given here. Such wavefronts are not plane wavefronts and the result (that they produce no sensor output) is not intuitively evident. However, it should be acknowledged that the analysis given evaluates the difference between the signals and does not indicate that the signal form is unchanged as a result of the existence of a negative region in the input function. One-dimensional simulations have indeed demonstrated that the data in the \pm diffraction orders is identical but also that the data in these orders identifies the presence of the negative region on the wavefront when compared to the intensity distribution produced by the unfiltered wavefront.

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2. When *a priori* information about the wavefronts to be measured is available (e.g. that it is likely to possess certain asymmetries or likely to have certain statistical properties) it is at least sometimes possible to optimise the filter function to improve algorithm performance. Insufficient simulation data has been obtained better to quantify this statement. However, there are strong indications that it is preferable to use axially-symmetric filter functions to produce phase diversity data unless there is clear *a priori* information that the input wavefront is likely to possess asymmetries and the preferred axes of those asymmetries are known in advance. Without such knowledge of the symmetry axes it is as easy to reduce the performance of the algorithm as to improve it.
3. Iterative algorithms based on the class of 'error reduction' algorithms (e.g. Gerchberg-Saxton algorithm, Fienup algorithm) converge well for data having an $\text{SNR} \geq 30$ and almost always fail to converge when the $\text{SNR} \leq 10$. For SNR values between these limits the algorithms sometime converge and sometimes stagnate. In all simulations undertaken the initial guess for the wavefront phase was taken to be that the input wavefront was flat – this helps to ensure that only data-driven structures appear within the wavefront reconstruction. No attempt has been made to optimise algorithm performance and it seems likely that a more sophisticated algorithm would achieve superior sensitivity.

Due to changes within the programme, notably the patent application, it was not feasible to conduct experimental work within the 12-month period of this agreement.

10. Publications

A conference paper was presented at CLEO 2003. This paper and the form of acknowledgement given, was cleared through EOARD in advance of publication. A copy of the slides presented was provided to EOARD with the 3rd progress report.

A second conference paper for presentation at the 4th International Workshop on Adaptive Optics for Industry and Medicine is planned for October 2003. The acknowledgement to EOARD will be given in the form approved for the previous conference presentation and the contents of the paper will be submitted to EOARD for approval in advance of publication.

A paper on the analysis made of the filter properties required in generalised phase diversity is in preparation and will be submitted to EOARD in advance of submission to the journal for editorial consideration.

11. Future work

Within the context of a programme on wavefront metrology funded by PPARC (the Particle Physics and Astronomy Research Council), we plan to continue work on the generalised phase diversity concept for commercial metrology applications and will provide to EOARD a supplementary report based on the PPARC programme at an appropriate time (probably about 1 year hence).

Aspects of possible future work of possible interest to EOARD and unlikely to be considered within this metrology programme would be

- optimisation of the diversity function in order to obtain best performance from the CAOS wavefront sensor using *a priori* knowledge of the statistical properties of atmospheric turbulence
- consideration of the use of multi-conjugate correction as a mechanism to control the effects of branch points in the wavefield
- experimental feedback control of an AO system using null wavefront sensors based on generalised phase diversity.

If required a 12-month programme to address these issues can be offered for approx \$25000.

Compact adaptive optics systems II (CAOS-II)**12. Compliance**

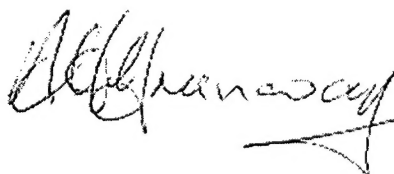
In accordance with the requirements of the Order under which this programme was granted, I certify that there was one subject inventions to declare as defined in FAR 52.227-13, during the performance of this programme. A copy of the UK patent application made in respect of this invention was provided to EOARD with the 3rd Progress Report. In discussions with respect to this invention, EOARD have indicated (e-mail from Beth Wann to Mike Cox dated 1 August 2003 at 14h18) that EOARD are prepared retrospectively to replace FAR 52.227-113 with FAR 52.227-12.

The contractor, Heriot-Watt University hereby declares that, to the best of its knowledge and belief, the technical data delivered herewith (including data provided with Progress Reports during execution of the programme) under Contract Number FA685-02-M4089 is complete, accurate and complies with all requirements of the contract.

The contractor, Heriot-Watt University, wished to reserve rights to use for other purposes (such as publication or proposal for future work) computer software and figures used in this report. However, right is granted to the Government to use this material freely for any Government purpose and without reference to the contractor.

Technical data	Basis for assertion	Asserted Rights category	Name of Person Asserting Restrictions
Figures 1, 2 and 3 in report text	Used in previous Heriot-Watt copyright documents	Government Purpose Rights	Prof A H Greenaway
Computer software used in simulations	Developed partially at private expense (Programme funded by UK PPARC Research Council)	Government Purpose Rights	Prof A H Greenaway

Dated: 22 September 2003.



Prof A H Greenaway.....